2DL Final Solutions

1. Double slit interference

$$d \sin\theta = \frac{\lambda}{2}$$

$$d_{best} = \frac{\lambda_{best}}{2\sin\theta_{best}}$$

$$= \frac{500\text{ nm}}{2\sin(6.5^\circ)}$$

$$= 2208.42\text{ nm}$$

$$\delta d = \sqrt{\left(\frac{\partial d}{\partial \lambda}\delta\lambda\right)^2 + \left(\frac{\partial d}{\partial \theta}\delta\theta\right)^2}$$

$$= \sqrt{\left(\frac{\delta\lambda}{2\sin\theta}\right)^2 + \left(\frac{\lambda\cos\theta}{2\sin^2\theta}\delta\theta\right)^2}$$

must be in radians

$$= 311.648\text{ nm}$$

$$\boxed{d = 2200 \pm 300\text{ nm}}$$

2. χ^2 fit

a)
$$\boxed{\chi^2 = \sum_{i=1}^{N} \frac{(y_i - f(x_i))^2}{\sigma_y^2}}$$

$$= \sum_{i=1}^{N} \frac{(y_i - Bx_i)^2}{\sigma_y^2}$$

b) Best fit value for B: Minimize χ^2 (error) with respect to B

$$\frac{\partial \chi^2}{\partial B} = 0$$

$$\frac{\partial \chi^2}{\partial B} = \sum_{i=1}^{N} \frac{2}{\sigma_y^2}(y_i - Bx_i)(-x_i)$$

$$0 = \sum_{i=1}^{N} x_i y_i - B\sum_{i=1}^{N} x_i^2$$

$$\boxed{B = \frac{\sum_{i=1}^{N} x_i y_i}{\sum_{i=1}^{N} x_i^2}}$$

3. Counting Number Problem

a) In book def'n of Poisson distribution, $P_\mu(\nu) = e^{-\mu}\frac{\mu^\nu}{\nu!}$, where ν is the number of counts in some time interval T and μ is the expected (average) number of counts in time T. In this problem, μ is given as an expected rate, so the expected # of counts in time T is μT.

$$\boxed{P_{\mu T}(\nu) = e^{-\mu T}\frac{(\mu T)^\nu}{\nu!}}$$

b) T = 4 min. ~~[illegible]~~ expected # counts = 4, $\nu = 4$: $P = e^{-4} \cdot 4^4/4! = 0.195$

c) Poisson → Gauss when expected # counts is large ($\mu T \gg 1$)

d) mean $\bar{X} = 81$, stdev $\sigma = \text{sqrt(mean)} = 9$

$$P = \frac{1}{9\sqrt{2\pi}}\int_{72}^{90} e^{-(x-81)^2/162}\,dx = \frac{1}{\sqrt{2\pi}}\int_{-1}^{1} e^{-z^2/2}\,dz$$

4. e/m_e

a) Weighted average w/ $\left(\frac{e}{m}\right) = \left(\frac{e}{m}\right) \pm \sigma_{e/m}$

$$\left(\frac{e}{m}\right)_{av} = \frac{\sum_i w_i \left(\frac{e}{m}\right)_i}{\sum_i w_i} \quad \text{where} \quad w_i = \frac{1}{\sigma_i^2}$$

$$\boxed{\left(\frac{e}{m}\right)_{av} = 1.75 \times 10^8 \ C/g}$$

b) χ^2

$E = 1.75 \times 10^8 \ C/g$

$$\chi^2 = \sum_K \frac{(O_K - E_K)^2}{\sigma_K^2}$$

$$= \frac{(1.76-1.75)^2}{0.2^2} + \frac{(1.80-1.75)^2}{0.4^2} + \frac{(1.36-1.75)^2}{0.9^2}$$

$$\boxed{\chi^2 = 0.21}$$

c) $d = n - c$

$n = 3$ (3 independent trials)

$c = 1$ (calculated expected value from data)

$\therefore d = 2$

$\tilde{\chi}^2 = \chi^2/d$

$$\boxed{\tilde{\chi}^2 = 0.11}$$

Data is consistent w/hypothesis that e/m is a constant since $\tilde{\chi}^2 \lesssim 1$.